What I have talked in Recitation 6: Review of what you have learned so far

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Disclaimer

- These slides are designed exclusively for students attending section 1, 2 and 3 for the course 640:244 in Fall 2013. The author is not responsible for consequences of other usages.
- These slides may suffer from errors. Please use them with your own discretion since debugging is beyond the author's ability.

First Order Linear ODEs

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- Please find examples in older slides.

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- Please find examples in older slides. Also make sure you learn how to check your answers.

Separable ODEs

- How it looks like

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Exact ODEs and those can-be-made-exact ODEs

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- So if the ODE above is exact, then one can express its implicit solution as
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So it suffices to recover the $\Psi(x,y)$ from $M(x,y)$ and $N(x,y)$.

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- Please find example problems in earlier slides.

2nd-order Linear Homogeneous ODE: Constant Coefficient

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Denote by r_1, r_2 the two roots.

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$$y(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}.$$

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- If $r_1 = r_2 = r$ (must be real), then the general solution is

$$y(t) = C_1 e^{rt} + C_2 t e^{rt}.$$

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- Principle of superposition: If functions $y_1(t), y_2(t)$ are solutions to this ODE,

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$$Ay_1(t) + By_2(t)$$

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for some number A, B.

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- In other words, $y_1(t), y_2(t)$ are linearly independent to each other

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  for some number A, B.
- In other words, $y_1(t), y_2(t)$ are linearly independent to each other and forms a fundamental set of solutions.

2nd-order Linear Homogeneous ODE: General Theory

- In addition, if the Wronskian

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  as a function of t, is not constantly 0, then ALL THE SOLUTIONS of this ODE looks like

  $$Ay_1(t) + By_2(t)$$

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- Then you get $v'(t)$ and by integration you get $v(t)$ and thus $y_2(t) = v_(t)y_1(t)$ and thus the general solution $y(t) = C_1y_1(t) + C_2y_2(t)$.

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